



**Poisson process**

exponential distribution

- memoryless property
- $\min(X1, X2)$  is exponential
- $P(X1 = \min(X1, X2))$

counting process

- stationary increment
- independent increment

first def

counting process with indep. Poisson increment

second def

- counting process with stationary and independent increments
- $P(N(h) = 1) = \lambda * h + o(h)$
- $P(N(h) \geq 2) = o(h)$

third def

- counting process
- interarrival times are i.i.d.  $\text{Exp}(\lambda)$

properties

- conditional distribution given arrival time
- superposition: independent  $N1(t) + N2(t)$  is Poisson process
- thinning
  - thinning with fixed prob
  - thinning with time-varying prob

$T1 | N(t) = 1$  is uniform on  $[0,t]$   
 $(S1, S2, \dots, Sn) | N(t) = n$  are equivalent in distribution to order statistics of  $n$  i.i.d. uniform on  $[0,t]$

extensions

- nonhomogeneous Poisson process
  - intensity function  $\lambda(t)$
- compound Poisson process
  - mean and variance calculation
- conditional Poisson process (not required)